# Market Efficiency: An Empirical Survey In Peru And Other Selected Countries

# Samuel Mongrut-Montalván

#### Resumen

Este artículo explica tres métodos para verificar la hipótesis de eficiencia de mercado en su forma débil: el método de coeficientes de autocorrelación, el método de ratio de varianza y el método de regresión sobre rezagos. Todos ellos tienen diferentes fortalezas y limitaciones, precisamente por ello proveen pistas complementarias sobre la eficiencia de los mercados de capitales. Los tres métodos se aplican a una muestra compuesta por índices bursátiles de once países y el índice mundial. Los resultados revelan la aparición de anomalías en los mercados de capitales, específicamente sobre-reacción y retorno a la media, con diferencias en su duración. Las diferencias en duración podrían ser debidas a diferencias en horizontes de inversión entre los inversionistas de los países considerados.

#### Abstract

This paper surveys three methods for testing the weak form of market efficiency: the autocorrelation coefficient, the variance ratio, and the <code>lead-on-the-lag</code> regression. All of them have different strengths and limitations, so they provide complementary insights about market efficiency. The three methods are applied to a sample of eleven countries and to the world stock index. The results reveal the appearance of market anomalies in capital markets, specifically overreaction and mean reversion, with differences in timing and duration. These differences could be due to cross-country differences in investment horizons among investors.

#### INTRODUCTION

During the last decade, there has been a revival of the old debate about market efficiency. This was due to the discovery of anomalies and reversal patterns in stock prices that could be seen as violations of the efficient market hypothesis. This paper deals with investors' overreaction and reversal patterns in stock prices. Investors seem to overreact to earnings announcements, and so they overvalue the stock attached to good announcements. However, sooner or later the stock returns will revert to the mean. Mean reversion therefore implies that stocks that are doing well today will perform poorly in future.

In particular, we are interested in answering the following questions: Do we see overreaction and mean reversion across countries? And related to this question, do these anomalies have the same features across different capital markets? In order to answer these questions, we use three different approaches that deal with the issues of random walk, mean reversion and predictable components in stock returns. These approaches are the autocorrelation coefficient, the variance ratio and the lead-on-the-lag regression, respectively. Although the three approaches seem to be quite different, they are not. They just scrutinize the same phenomenon from different points of view.

The autocorrelation coefficient approach was one of the earliest approaches to test whether the random walk hypothesis in its weakest version could be rejected or not. If we can reject it, the efficient market hypothesis does not hold because stock returns may—to some extent—be accounted for by past returns, an impossible explanation in an efficient market.

Due to the fact that the autocorrelation coefficient approach is not suitable for long-term horizons (it loses degrees of freedom), the variance ratio approach aims at testing the random walk hypothesis in the long run. If the stock returns have significant transitory (predictable) components, the stock returns will exhibit negative autocorrelation (the variance ratio will fall below one) as the lag (q) horizon return increases. In fact, this is also a test for mean reversion. Therefore the violation of the random walk hypothesis would imply the presence of mean reversion.

The *lead-on-the-lag* regression approach aims at discovering the presence of transitory and permanent (random walk) components in stock returns. Rejecting the random walk hypothesis will favour a mean reversion process in the long run, but eventually this mean reversion (due to transitory components) will be dominated by the permanent component (random walk) as the lag (q) horizon return increases.

All three approaches are linear models and are related to the weak version of the market efficiency hypothesis, but each approach has its particular advantages and limitations. Their main and shared limitation is the lack of reliable long-run estimates. Although the implications of rejecting the null hypothesis under each approach are different, the three approaches and their results are interconnected. One could argue that going from the autocorrelation coefficient approach to the *lead-on-the-lag* regression approach is just a way of getting deeper insights about market efficiency.

As we will see, the empirical findings match the results of the available literature with some minor differences in the third approach. The sample consists basically of the value-weighted stock market index for ten different countries and the world index. However, we also analyze the Peruvian value-weighted stock market index for the variance ratio approach.

The paper is organized as follows. The next section deals with some basic concepts, while the following three sections deal successively with one of the three different approaches. Each section addresses the theoretical background of the approach, its limitations and the empirical evidence found. The last section offers conclusions.

# 1. WHAT IS A RANDOM WALK MODEL?

In general, investors, whether individual or institutional, want to outperform or beat the market. However, for a long time, this has been considered and impossible goal because the best forecast of tomorrow's price, given a historical series, is today's price. Accordingly, the *fair game* principle underlying any efficient capital market implies that, any attempt to forecast stock prices is useless<sup>1</sup>. When researchers looked at the behaviour of stock prices, they saw an explosive behaviour whose underlying stochastic process remained unknown, and they tried to determine whether a stationary time series could bring higher predictability.

The simplest way to try to get a stationary time series is by looking at its first difference. For example, with the natural logarithm of the stock price at time "t" (pt) and at time "t-1" (pt-1) it is easy to establish the following relationship, which is in fact the stock's continuous return<sup>2</sup>.

<sup>1.</sup> The idea of a fair game is called the 'Martingale hypothesis' in financial jargon.

<sup>2.</sup> We may use continuous returns for two main reasons: to be consistent with the continuous capitalization of stock returns and to avoid possible bias in simple averages.

$$p_t - p_{t-1} = \varepsilon_t$$

Unfortunately, the first price difference is random and unpredictable because we do not know the true stochastic process that generates the returns. This impossibility to predict is the main statement of the random walk hypothesis. This model is the simplest version of a random walk model and it offers better features to be studied by means of econometric tools<sup>3</sup>. More technically, a random walk model is a special case of distributed lag models.

Trends in stock market indices are well documented, therefore we can include this trend or drift  $(\mu)$  in the model and solve for today's price.

$$p_t = \mu + p_{t-1} + \varepsilon_t \tag{1}$$

Depending on the assumptions made on the error term ( $\epsilon$ ), we may distinguish three main types of random walk models (Campbell; Lo and MacKinlay, 1997)<sup>4</sup>:

- Random Walk 1 (RW1): The simplest, but also the more restrictive version. It assumes
  that the errors are identical and Normal distributed through time with mean zero
  and constant variance (σ²). In other words, the errors are white noise. Furthermore, it
  assumes that the errors are independent (uncorrelated) in the linear and in the nonlinear sense.
- Random Walk 2 (RW2): It assumes that the errors are uncorrelated in a linear and non-linear way, but they are not identically distributed through time and so error variance is no longer a constant.
- Random Walk 3 (RW3): The assumption about independent errors is relaxed; therefore, although the errors will be linearly uncorrelated, they will be dependent in a nonlinear way (higher moment dependence).

As we can see, the less restrictive form of the random walk model is RW3 and is in fact one of the most common tests performed because it conveys the more realistic assumptions under the weak form of market efficiency. The random walk hypothesis is usually tested in the first and/or third versions of the model. This is because it is difficult to test for the RW2 due to the different distribution of errors through time. In this paper we only deal with the first and third versions of the random walk model (i.e. RW1 and RW3). Specifically, the autocorrelation coefficient approach and the variance ratio approach deal with the RW1,

<sup>3.</sup> Besides, the investor is usually more interested in the stock return rather than in its price.

<sup>4.</sup> Campbell; Lo and MacKinlay (1997) introduced this classification.

while the *lead-on-the-lag* regression approach deals with the RW3. However, it is important to notice that the variance ratio approach could also deal with a model specification corresponding to RW3<sup>5</sup>.

#### 2. THE AUTOCORRELATION COEFFICIENT APPROACH

#### 2.1 Theoretical framework

In this section we test for RW1. First, we must derive a test statistic that allow us to test the following null hypothesis:

$$H_o: \rho_{(a)} = 0$$

If we reject this null hypothesis, we face positive or negative "q" order autocorrelation. Let's consider the following AR (1) model:

(2)

Where "r" is the continuous return, "p" is the first-order autocorrelation coefficient (-1 ) and the error is white noise. Now, we need to derive a test statistic to test the null hypothesis. In order to do this, we first derive the variance for the return in period 't'. By repeated substitution in expression 2 and taking variance to both sides we get the variance for "<math>rt":

$$r_{t} = \rho(\rho.r_{t-2} + \varepsilon_{t-1}) + \varepsilon_{t} = \varepsilon_{t} + \rho\varepsilon_{t-1} + \rho^{2}r_{t-2} = \sum_{i=0}^{t} \rho^{i}\varepsilon_{i}$$

$$\sigma^{2}(r_{t}) = \sigma_{\varepsilon}^{2} \sum_{i=0}^{t} \rho^{2i} \approx \frac{\sigma_{\varepsilon}^{2}}{1 - \rho^{2}}$$
(3)

The next step is to obtain the first-order autocorrelation estimator and its mean and variance. The first-order autocorrelation coefficient in equation 2 and its first two moments are given by the following expressions<sup>6</sup>:

$$\hat{\rho(1)} = \frac{\sum_{t=1}^{n} \left( r_{t} - r \right) \left( r_{t-1} - r \right)}{\sum_{t=1}^{n} \left( r_{t-1} - r \right)^{2}} = \frac{\sum_{t=1}^{n} r_{t} r_{t-1}}{\sum_{t=1}^{n} r_{t}^{2} r_{t-1}} = \frac{\sum_{t=1}^{n} \left( \rho \cdot r_{t-1} + \varepsilon_{t} \right) r_{t-1}}{\sum_{t=1}^{n} r_{t-1}^{2}} = \rho + \frac{\sum_{t=1}^{n} \varepsilon_{t} r_{t-1}}{\sum_{t=1}^{n} r_{t-1}^{2}} \tag{4}$$

 $r_t = \rho.r_{t-1} + \varepsilon_t$ 

<sup>5.</sup> Campbell; Lo, and MacKinlay (1997) have shown the properties of such specification.

<sup>6.</sup> Equation 4 is the least square estimator of the first-order autocorrelation coefficient. Besides, note that expression 6 is obtained using expression 3.

$$E\left[\hat{\rho(1)}\right] = \rho \tag{5}$$

$$\sigma^{2} \left[ \rho(1) \right] = \frac{\sigma_{\varepsilon}^{2}}{\sum\limits_{t=1}^{n} r_{t-1}^{2}} = \frac{\sigma_{\varepsilon}^{2}}{n \left( \sigma^{2}(r_{t-1}) \right)} = \frac{\sigma_{\varepsilon}^{2}}{n \left( \frac{\sigma_{\varepsilon}^{2}}{1 - \rho^{2}} \right)} = \frac{1 - \rho^{2}}{n}$$
(6)

As a last step, we only need to standardize the first order autocorrelation estimator to obtain a test statistic to test the null hypothesis. The previous equations yields:

Under the null hypothesis,  $\rho_{(1)} = 0$ , the previous test simplifies to the following expression:

(7)

This test can be generalized to a higher order autocorrelation. We know that the square of the normal standard variable in expression 7 produces a chi-square variable with one degree of freedom. Besides, the addition of "q" independent chi-square variables with one degree of freedom generates the Box and Pierce Q-statistic<sup>7</sup>:

$$Q_q = (n) \sum_{t=1}^{q} \hat{\rho_t^2} \approx X_{(q)}^2$$
 (8)

Furthermore, Ljung and Box (1978) provide the following modified "*Q-statistic*" for small sample size that aims at correcting for explosive behaviour in the time series:

$$Q_m^* = (n)(n+2) \sum_{q=1}^m \frac{\rho^2(q)}{n-q} \approx X_{(m)}^2$$
 (9)

The problem now is to determine the exact number of autocorrelations, because if there are few, we may not detect the presence of higher-order autocorrelation and if there are too many, the test may not have enough statistical power due to negligible high-order autocorrelations<sup>8</sup>.

<sup>7.</sup> This expression is the basis of the so-called "Portmanteau Statistics" (Campbell; Lo and MacKinlay 1997).

Note that errors are assumed to have a constant variance.

#### 2.2 Empirical evidence

Table 1 (A, B and C) shows the empirical evidence obtained from 10 countries and the world monthly value-weighted stock market index. The data corresponds to the value-weighted stock market index of 10 countries plus the world index for the period 01.01.1973 – 30.09.1999. The table shows the autocorrelations' coefficients until the fifth period (q = 5) and the Ljung and Box "Q-statistic" with its 'p-value' for the whole period and two subperiods<sup>9</sup>. The general rule is that one cannot reject the null hypothesis; therefore there is no statistical evidence against RW1. However, there are some countries where one finds significant positive serial correlation at 90%, for example Switzerland (autocorrelations 3 and 4), Germany (autocorrelations 3 and 4), Italy (autocorrelations 1, 3 and 4) and Great Britain (autocorrelation 3).

Nevertheless, in all cases the magnitude of this positive serial correlation and its explanatory power is small. For example, one of the highest and highly significant positive serial correlations found was for Italy in the first lag-period ( $\rho_{(j)}=0.18$ ). However, this implies that 3,2% of the variation in the monthly value-weighted index is predictable using the previous monthly index return<sup>10</sup>. We also finds evidence of negative autocorrelation. For example, for Great Britain (autocorrelations 2 and 5) and Italy (autocorrelation 2), but its explanatory power is smaller than for positive autocorrelation coefficients.

In general, one may expect short-run positive autocorrelation of small magnitude between monthly stock index returns. Campbell; Lo and MacKinlay (1997) studied the daily, weekly and monthly stock index (equal-weighted and value-weighted) using the data from the Centre for Research in Security Prices (CRSP) and they found the same result. Furthermore, the order of magnitude of the mean and the standard deviation found by Campbell; Lo and MacKinlay (1997) for the value-weighted index are similar to those shown in table 1.

#### 3. THE VARIANCE RATIO APPROACH

# 3.1 Theoretical framework

In this section we use the variance ratio approach to examine if stock prices show long-run mean-reverting patterns. The variance ratio approach basically relies on the fact that if stock returns follow a random walk, the return variance grows linearly with time interval

<sup>9.</sup> Information was obtained from 'DataStream'.

The R2 of a regression of the monthly index returns on a constant and its first lag is equal to the square of the regression's slope: (0,18) 2 = 3,2%.

Table 1A Autocorrelation in Monthly Value-Weighted Market Stock Index the World and Various Countries Different Periods

					Autocorrelati	Autocorrelation Coefficients		
Sample Period	Observ.	Mean	S.D	p1	p2	р3	p4	p5
1. World V.W.Index								
. 1973/2002-1999/2009	320	26'0	4,18	90'0	-0'05	0,02	-0,04	0'0
Ljung-Box (p-value)				0,33	0,41	09'0	69'0	95'0
. 1973/2002-1986/2005	160	0,91	4,10	0,13	-0,01	0,11	20'0	60'0
Ljung-Box (p-value)				0,11	0,28	0,20	0,24	0,22
. 1986/2006-1999/2009	160	1,02	4,26	-0,01	60'0-	80'0-	-0,15	0'03
Ljung-Box (p-value)				06'0	0,53	0,52	0,21	0,31
2. RLCA V.W. Index								
. 1973/2002-1999/2009	320	0,83	5,04	00'0	90'0-	80'0	-0'0	90'0
Ljung-Box (p-value)				86'0	0,52	0,31	0,26	0,28
. 1973/2002-1986/2005	160	0,85	5,56	-0,01	80'0-	0,15	0,02	0,14
3. RLAU V.W. Index								
. 1973/2002-1999/2009	320	26'0	6,49	0,01	90'0-	-0,02	90'0	-0'02
Ljung-Box (p-value)				0,88	0,54	0,72	0,65	0,67
. 1973/2002-1986/2005	160	1,03	6,82	90'0	80'0-	0'03	0,10	0,02
Ljung-Box (p-value)				0,45	0,45	0,63	0,50	0,63
. 1986/2006-1999/2009	160	06'0	6,15	-0'02	-0'03	90'0-	0,01	-0,14
Ljung-Box (p-value)				0,51	0,74	92'0	88'0	0,51
4. RLSW V.W. Index								
. 1973/2002-1999/2009	320	0,83	5,04	60'0	80'0-	0,04	90'0-	90'0
Ljung-Box (p-value)				0,11	0,10	0,17	0,19	0,20
. 1973/2002-1986/2005	160	95'0	4,41	0,03	-0,14	0,16	0,05	-0,01
Ljung-Box (p-value)				89'0	0,17	90'0	60'0	0,15
. 1986/2006-1999/2009	160	1,10	2,60	0,12	-0'02	-0'03	-0,12	80'0
Ljung-Box (p-value)				0,13	0,27	0,42	0,25	0,28

Table 1B
Autocorrelation in Monthly Value–Weighted Market Stock Index the World and Various Countries (Continuation)
Different periods

					Autocorrelation	Autocorrelation Coefficients		
Sample Period	Observ.	Mean	S.D	p1	p2	р3	p4	p5
5. RLFR V. W. Index								
. 1973/2002-1999/2009	320	1,13	6,17	80'0	90'0-	90'0	0,02	0,01
Ljung-Box (p-value)				0,16	0,21	0,22	0,34	0,47
. 1973/2002-1986/2005	160	1,23	6,40	0,10	90'0-	20'0	0,04	0,04
Ljung-Box (p-value)				0,22	98'0	0,40	0,51	0,61
. 1986/2006-1999/2009	160	1,03	5,95	90'0	-0'05	90'0	-0,05	-0,04
Ljung-Box (p-value)				0,46	0,63	69'0	0,75	0,83
6. RLGE V. W. Index								
. 1973/2002-1999/2009	320	0,85	5,35	0,04	-0,02	0,04	0,04	60'0-
Ljung-Box (p-value)				0,45	0,71	0,77	62'0	0,53
. 1973/2002-1986/2005	160	0,92	4,41	-0,01	-0,04	0,24	0,10	90'0-
Ljung-Box (p-value)				0,95	0,88	0,02	0,02	0,04
. 1986/2006-1999/2009	160	62'0	6,17	90'0	-0,02	-0,04	-0,01	-0,11
Ljung-Box (p-value)				0,43	0,71	0,83	0,92	69'0
7. RLIT V.W. Index								
. 1973/2002-1999/2009	320	1,10	7,35	0,07	00'0	90'0	0'02	-0,01
Ljung-Box (p-value)				0,19	0,42	0,40	0,45	09'0
. 1973/2002-1986/2005	160	1,57	7,65	0,18	-0,01	0,12	90'0	0,01
Ljung-Box (p-value)				0,03	80'0	90'0	60'0	0,16
. 1986/2006-1999/2009	160	0,63	7,02	-0,05	0,02	0,03	0,03	-0,07
Ljung-Box (p-value)				05'0	0,77	0,88	0,94	06'0
8. RLIP V.W. Index								
. 1973/2002-1999/2009	320	0,54	5,38	0,02	00'0	0,01	0,03	0,02
Ljung-Box (p-value)				0,78	96'0	66'0	86'0	66'0
. 1973/2002-1986/2005	160	0,88	4,28	-0,01	0,03	0,01	-0,04	-0,15
Ljung-Box (p-value)				0,88	06'0	86'0	86'0	0,51
. 1986/2006-1999/2009	160	0,19	6,28	0,02	-0,02	-0,01	0,04	80'0
Ljung-Box (p-value)				62'0	0,93	86'0	86'0	0,92

Autocorrelation in Monthly Value-Weighted Market Stock Index the World and Various Countries (Continuation) Different periods Table 1C

Sample Period	i							
	Observ.	Mean	S.D	p1	p2	p3	p4	p5
9. RLNL V. W. Index								
. 1973/2002-1999/2009	320	1,20	2,00	80'0	00'0	0,01	90'0-	0'03
Ljung-Box (p-value)				0,17	62'0	0,59	0,55	0,64
. 1973/2002-1986/2005	160	1,13	5,03	0,04	00'0	90'0	-0,02	0,13
Ljung-Box (p-value)				0,62	0,88	0,84	0,93	09'0
. 1986/2006-1999/2009	160	1,26	4,98	0,12	0,01	-0,05	80'0-	60'0-
Ljung-Box (p-value)				0,14	0,34	0,47	0,47	0,43
10. RLUK V. W. Index								
. 1973/2002-1999/2009	320	1,20	6,16	80'0	-0,10	90'0	0,04	-0,12
Ljung-Box (p-value)				0,13	0,07	60'0	0,14	0,04
. 1973/2002-1986/2005	160	1,32	7,13	0,11	60'0-	0,13	90'0	-0,15
Ljung-Box (p-value)				0,18	0,20	0,12	0,17	0,07
. 1986/2006-1999/2009	160	1,09	5,03	0,04	-0,12	60'0-	0,02	-0'02
Ljung-Box (p-value)				0,61	0,25	0,24	0,38	0,47
11. RLUS V. W Index								
. 1973/2002-1999/2009	320	1,04	4,48	00'0	-0'03	0,01	-0,04	60'0
Ljung-Box (p-value)				86'0	06'0	76'0	0,94	0,65
. 1973/2002-1986/2005	160	08'0	4,52	0,01	-0,01	80'0	60'0	0,12
Ljung-Box (p-value)				68'0	66'0	0,81	69'0	0,46
. 1986/2006-1999/2009	160	1,28	4,44	-0,02	-0,04	80'0-	-0,16	0,04
Ljung-Box (p-value)				62'0	0,83	0,71	0,25	0,34
Legend: World V.W: Returns of the World value-weighted index RISW V.W: Returns of the Swiss value-weighted index	ed index ed index	RLCA V.W: Returns RLFR V.W: Returns	of the Canadian v	RICA V.W.: Returns of the Canadian value-weighted index RIFR V.W.: Returns of the French value-weighted index		RLAUV.W: Returns of the Austrian value-weighted index RLGEV.W: Returns of the German value-weighted index	ustrian value-wei erman value-weig	ghted index inted index
RLITV.W: Returns of the Italian value-weighted index	d index	RLP V.W: Returns	of the Japanese va	RLPV.W. Returns of the Japanese value-weighted index		RLNL V.W: Returns of the Dutch value-weighted index	utch value-weigh	ted index

"q". This implies that for example the two-year return variance must be twice the one-year return variance. In general, let's consider the variance of the one-period return and the variance of the "q" period returns:

$$\sigma^2 (p_t - p_{t-1}) = \sigma^2 (r_1) = \sigma^2 (\mu + \varepsilon_t) = \sigma_\varepsilon^2$$

$$\sigma^2 \left( p_t - p_{t-q} \right) = \sigma^2 \left( r_q \right) = \sigma^2 \left( \mu + p_{t-1} + \varepsilon_t - p_{t-q} \right)$$

By recursive substitution in the equation for the "q" period return we get the relationship between the one-period return and the "q" period return:

$$\sigma^{2}(r_{q}) = \sigma^{2}(q\mu + \varepsilon_{t-q} + \varepsilon_{t-q+1} + \dots + \varepsilon_{t}) = q\sigma_{\varepsilon}^{2}$$

But from the equation of the one-period return, we know that the variance of the residual is equal to the variance of the one-period return, hence:

$$\sigma^2(r_q) = q\sigma^2(r_1) \tag{10}$$

From expression 10, we can easily derive the variance ratio for the q-period return:

$$VR(q) = \frac{\sigma^2(r_q)}{(q)\sigma^2(r_1)} = 1$$
 (11)

Under the null hypothesis of the random walk, the variance ratio should equal one. It predicts that if stock returns show a negative serial correlation, it will be smaller than one, while with a positive serial correlation, it will be larger than one. Again, we are interested in constructing a test statistic to test for random walk hypothesis, specifically RW1. Let's start by defining the mean and the variance of non-overlapping stock returns (Lo and MacKinlay 1999):

$$\hat{\mu} = \frac{1}{nq} \sum_{t=1}^{nq} (p_t - p_{t-1})$$
 (12)

$$\hat{\sigma^{2}(r_{t})} = \frac{1}{nq} \sum_{t=1}^{nq} \left( p_{t} - p_{t-1} - \hat{\mu} \right)^{2}$$
 (13)

$$\hat{\sigma^{2}(r_{q})} = \frac{1}{nq} \sum_{t=1}^{n} \left( p_{qt} - p_{qt-q} - q \hat{\mu} \right)^{2}$$
 (14)

Equation 12 represents the population mean of one-period returns, while equations 13 and 14 represent the variance of the one-period returns and "q" period returns assuming non-overlapping returns. If one does not allow for overlapping returns, the number of observations will decrease as the time interval "q" increases. Therefore, it is a good idea to allow for overlapping observations. If one allows for overlapping observations, equation 14 must be written in the following way:

$$\sigma^{2}(r_{q}) = \frac{1}{nq^{2}} \sum_{t=q}^{nq} \left( p_{t} - p_{t-q} - q \hat{\mu} \right)^{2}$$
 (15)

This estimator contains (nq-q+1) observations, whereas equation 14 only has 'n' non-overlapping observations<sup>11</sup>. This feature of overlapping observations creates a problem: artificial serial correlation. Furthermore, since formulas 13 and 15 are for populations and we work with a sample, we need to adjust the variances of the one-period return and q-period overlapping return to get unbiased estimators. In order to solve both problems, Campbell; Lo and MacKinlay (1997) propose the following efficient estimators for the variance of the one-period returns and q-period overlapping returns:

$$\sigma_{ef}^{2}(r_{t}) = \frac{1}{nq - 1} \sum_{t=1}^{nq} \left( p_{t} - p_{t-1} - \mu \right)^{2}$$
(16)

$$\sigma_{ef}^{2}(r_{q}) = \frac{1}{q(nq - q + 1)\left(1 - \frac{q}{nq}\right)} \sum_{t=q}^{nq} \left(p_{t} - p_{t-q} - q\hat{\mu}\right)^{2}$$
(17)

From the previous exposition, an efficient variance ratio statistic will be given by the following expression:

$$VR_{ef}(q) = \frac{\sigma_{ef}^{2}(r_{q})}{\sigma_{ef}^{2}(r_{t})}$$
(18)

Campbell; Lo and MacKinlay (1997) establish the following test-statistic of the efficient variance ratio under the null hypothesis of RW1:

<sup>11.</sup> The total number of observations is 'ng'.

(19)

This formula holds that the efficient variance ratio distributes asymptotically like the normal standard distribution (Z).

# 3.2 Empirical evidence

In this subsection, using expression 19, we test whether the random walk hypothesis implied by RW1 and the mean reversion hypothesis can be reliably accepted from the data. The empirical evidence is divided in two major parts: Peruvian evidence of the daily value-weighted stock index return and world evidence of the monthly value-weighted stock index returns.

This division of the empirical evidence allows us to compare the results with two other former studies about the variance ratio. The first part will be compared with the study by Campbell; Lo and MacKinlay (1997) and the second part will be compared with the study by Poterba and Summers (1988).

# 3.2.1 Empirical evidence from the Peruvian Index

Table 2 shows the variance ratios statistics for the Peruvian daily value-weighted stock index return. This index belongs to the Lima Stock Exchange for the period 01.01.92 to 31.12.97. The variation of the index reflects continuous compounding returns<sup>12</sup>. An interesting observation is the positive and highly significant autocorrelation for short periods lags. In particular, the results seem robust for the first four equivalent months at 90% level of confidence. However, for longer horizons the test statistics loses its power and although we see a variance ratio below one for the second year (24 equivalent months), this result is not statistically significant.

The index is expressed in dollars and adjusted by dividends. The information was obtained from the 'Economática'
database.

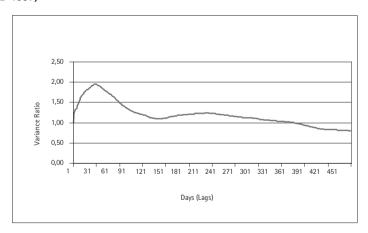
Table 2
Variance Ratios for Daily Stock Index Returns Value-Weighted
Peruvian Index: 1992/01-1997/12<sup>1/</sup>

Sample Period	Observ. "ng"		Nu	mber "q"	(lag days	s) to form	n the vari	ance rati	io
Days		10	20	40	60	80	120	240	480
Equivalent Weeks		2	4	8	12	16	24	48	96
Equivalent Months		0,5	1	2	3	4	6	12	24
Variance ratios Test Statistics	1.343	1,51 <i>5,55</i>	1,77 <i>5,66</i>	1,95 <i>4,86</i>	1,75 <i>3,12</i>	1,50 <i>1,80</i>	1,21 0,60	1,23 0,48	0,81 -0,27

<sup>1/:</sup> Test statistics in italics are significant at 90% level of confidence. The value from the Z-table is 1,645 for 90% of confidence.

The results strongly reject the RW1 hypothesis in the first four equivalent months, but they cannot reject it for longer horizons. Figure 1 shows the typical pattern of the variance ratio. (See appendix). In this sense, it shows a sort of overreaction in the short run and a slow adjustment in the long run.

Figure 1 Variance Ratios of the Peruvian Value-Weighted Stock Market Index (1992-1997)



These results are different from the results obtained by Campbell; Lo and MacKinlay (1997). They used weekly instead of daily stock index returns and they also found variance ratios above one, that were however statistically negligible, so they didn't reject the RW3 hypothesis<sup>13</sup>.

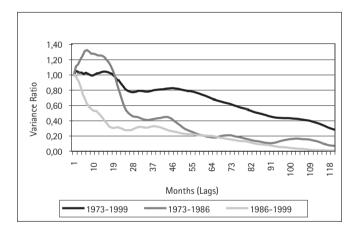
<sup>13.</sup> Their time span was considerably larger, because it covered from 1962 to 1994 of the stock index returns of the CRSP.

# 3.2.2 Empirical evidence from other indexes

In general, table 3 (A, B and C) shows evidence in favour of the RW1. However, this could represent the weak power of the variance ratio test for long run horizons, at least for the RW1 hypothesis.

At first glance, we may conclude that the null hypothesis of the presence of RW1 cannot be rejected. However, there are some notable exceptions worth discussing. The first concerns the world-wide index. Though we do not see statistical evidence against RW1, the variance ratio falls below one in the first and second years from 1986 to1999 and this result is statistically significant. Furthermore, if one looks at Figure 2, the pattern of the variance ratio curve for this period shows a negative serial correlation.

Figure 2: Variance Ratios of the World Value Weighted Stock Market Index



The second interesting result concerns some countries like Austria, Great Britain and Italy. The results of the first two countries corroborate the negative serial correlation during the first three years and are statistically significant for the second sub-period (Austria) and for the whole period (Great Britain). The results for Italy for the 3<sup>rd</sup> and 12<sup>th</sup> months are consistent with the results in the previous section about short-run positive serial correlation. According to results in the previous section there is a short-run overreaction (up to the fourth month). The variance ratio approach lets us exceed this horizon to find evidence for mean reversion in the case of the world index (i.e. twelve-month), but for longer periods the test loses its statistical power, so one cannot test for mean reversion in the long run.

Table 3A Variance Ratios for Monthly Stock Index Returns the World and Different Countries

2012	COSEIV.			Numbe	r q (lags mon	iths) of base o	oservations a	gregated to to	Number "q" (lags months) of base observations aggregated to form the variance ratio	ce ratio			
	"bu"	က	9	12	24	36	48	09	72	84	96	108	120
1. World V.W.Index													
. 1973/2002-1999/2009	320	1,03	1,03	1,03	0,81	62'0	0,82	0,73	0,62	0,51	0,44	0,40	0,29
Test Statistics		0,38	0,20	0,12	-0,60	-0,55	-0,41	-0,54	-0,70	-0,84	06'0-	-0,89	-1,02
. 1973/2002-1986/2005	160	1,14	1,31	1,26	0,59	0,41	0,38	0,21	0,21	0,14	0,14	0,16	0'0
Test Statistics		1,22	1,60	98'0	-0,94	-1,09	-1,00	-1,14	-1,03	-1,04	76'0-	06'0-	-0,93
. 1986/2006-1999/2009	160	0,91	89'0	0,50	0,28	0,32	0,25	0,21	0,16	0,10	0'02	0,02	0,0
Test Statistics		-0,73	-1,63	-1,69	-1,67	-1,26	-1,20	-1,14	-1,10	-1,08	-1,07	-1,04	-1,00
2. RLCA V.W. Index													
. 1973/2002-1999/2009	320	0,95	96'0	1,00	0,75	0,62	0,49	0,41	0,28	0,22	0,19	0,20	0,15
Test Statistics		-0,57	-0,31	0,003	-0,83	-1,00	-1,17	-1,20	-1,32	-1,33	-1,28	-1,20	-1,20
. 1973/2002-1986/2005	160	0,92	1,04	1,23	26'0	0,72	0,50	0,32	0,14	0'02	0'0	60'0	0'0
Test Statistics		-0,71	0,23	0,78	-0,07	-0,52	-0,80	86'0-	-1,12	-1,14	-1,05	-0,97	66'0-
. 1986/2006-1999/2009	160	66'0	0,81	0,61	0,32	0,37	0,33	0,28	0,22	0,13	80'0	0,04	0,0
Test Statistics		-0,12	-1,00	-1,33	-1,57	-1,17	-1,08	-1,03	-1,01	-1,04	-1,04	-1,01	66'0-
3. RLAU V.W. Index													
. 1973/2002-1999/2009	320	0,95	0,92	0,85	0,62	0,44	0,37	0,32	0,24	0,16	0,16	0,20	0,16
Test Statistics		-0,54	-0,61	69'0-	-1,25	-1,47	-1,42	-1,38	-1,41	-1,43	-1,33	-1,20	-1,19
. 1973/2002-1986/2005	160	1,00	1,04	1,12	0,78	0,44	0,28	0,19	0,14	0,05	90'0	80'0	0'02
Test Statistics		-0'03	0,19	0,41	-0,50	-1,04	-1,15	-1,17	-1,12	-1,15	-1,06	-0,97	96'0-
. 1986/2006-1999/2009	160	0,88	0,75	0,43	0,11	0,11	90'0	0,04	0,03	0,02	0,02	0,02	0,0
Test Statistics		86'0-	-1,30	-1,93	-2,06	-1,66	-1,51	-1,37	-1,26	-1,18	-1,10	-1,04	-1,00
4. RLSW V.W. Index													
. 1973/2002-1999/2009	320	1,05	1,04	1,06	68'0	0,78	0,71	69'0	0,62	0,54	0,46	0,38	0,29
Test Statistics		99'0	0,32	0,28	-0,37	-0,57	99'0-	-0,63	-0,70	-0,78	98'0-	-0,93	-1,01
. 1973/2002-1986/2005	160	0,93	1,02	1,09	0,74	0,65	0,48	0,32	0,22	0,19	0,18	0,18	0,10
Test Statistics		-0'63	80'0	0,32	-0,60	99'0-	-0,83	86'0-	-1,01	86'0-	-0,92	-0,88	-0,91
. 1986/2006-1999/2009	160	1,10	1,02	0,94	0,62	0,47	0,39	0,34	0,30	0,20	0,16	0,10	90'0
1 1 1 1 1 1 1 1 E		1											

\* Test Statistics in bold are significant at 90% level of confidence

Table 3B Variance Ratios for Monthly Stock Index Returns the World and Different Countries (Continuation)

S. RLFR V. W. Index         "nq"         3         6         12           1.973/2002-1999/2009         320         1,06         1,12         1,08           Test Statistics         0,70         0,88         0,39           1.973/2002-1996/2005         160         1,07         1,15         1,08           Test Statistics         0,58         0,75         0,26           1.986/2006-1999/2009         160         1,03         1,02         0,88           1.973/2002-1999/2009         160         1,04         1,06         1,29           1.973/2002-1996/2005         160         0,93         1,16         1,29           1.973/2002-1996/2005         160         1,05         0,98         0,98           1.973/2002-1996/2005         160         1,05         0,98         0,98           1.973/2002-1996/2005         160         1,05         0,98         0,98           1.973/2002-1996/2005         160         1,20         1,29         1,54           1.973/2002-1996/2005         160         1,20         1,29         1,54           1.973/2002-1996/2005         160         0,92         0,94         0,92           1.973/2002-1996/2009         160         0,92 <th></th> <th>ו מווסכן א (נמלס וווסוונווס) כו סמכר אמנוסווס מללו כלמ ניס וסוווו נוור אמוומוובר ומנוס</th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th>		ו מווסכן א (נמלס וווסוונווס) כו סמכר אמנוסווס מללו כלמ ניס וסוווו נוור אמוומוובר ומנוס							
320 1,06 1,12 0,88 0,70 0,88 0,75 0,58 0,75 0,58 0,75 0,75 0,75 0,75 0,75 0,75 0,75 0,75		24	36 4	48 (	60 72	84	96	108	120
320 1,06 1,12 0,70 0,88 0,70 0,88 1,02 0,75 1,03 1,02 0,25 0,11 0,25 0,11 0,43 0,43 1,16 0,93 1,16 0,43 0,43 1,06 0,98 1,06 0,98 1,07 0,129 1,09 1,18 1,04 1,31 1,04 1,31 1,00 1,29 1,00 1									
0,70 0,88 (9,70 1,15 (1,		0,82	0,68 0,64		0,50 0,45	0,40	0,40	96'0	0,27
160 1,07 1,15 0,58 0,75 1,02 1,03 1,02 1,02 1,03 1,02 1,02 1,04 1,06 1,16 0,93 1,16 0,43 0,43 1,16 0,43 1,16 1,05 1,106 1,05 0,98 1,06 1,09 1,18 1,04 1,31 1,00 1,20 1,20 1,20 1,20 1,20 1,20 1,2		-0,58	-0,83 -0,82		-1,02 -1,02	-1,02	-0,95	-0,95	-1,04
0,58 0,75 0,58 0,75 0,00 0,25 0,11 0,00 0,25 0,11 0,00 0,25 0,11 0,00 0,43 0,43 0,43 0,43 0,43 0,44 0,43 0,44 0,40 0,44 0,40 0,90 0,90 0,90 0,90		0,82	0,71 0,52		0,25 0,20	0,21	0,24	0,18	80'0
160 1,03 1,02		-0,43	-0,53 -0,77		-1,07 -1,05	96'0-	98'0-	-0,87	-0'93
320 1,04 1,06 0,43 0,43 1,16 0,43 1,16 0,93 1,16 0,98 1,16 0,98 1,16 0,43 0,44 1,16 1,09 1,18 1,04 1,31 1,60 1,20 1,20 1,50 1,50 1,60 1,60 1,60 1,60 1,60 1,60 1,60 1,6		0,49	0,33 0,23		0,15 0,15	60'0	0'02	0'02	0,04
320 1,04 1,06 0,43 0,43 0,43 1,16 0,93 1,16 0,98 1,16 0,98 1,16 0,43 0,44 0,43 0,44 1,16 1,09 1,18 1,04 1,31 1,60 1,20 1,20 1,20 1,50 1,60 0,92 0,94 0,71 0,71 0,31 0,71 0,31 0,31 0,93 0,93 0,93 0,93 0,93 0,93 0,93 0,93		-1,17	-1,24 -1,24		-1,22 -1,10	-1,10	-1,05	-1,01	96'0-
320 1,04 1,06 0,43 0,43 1,16 -0,55 0,84 160 1,05 0,98 1,04 1,18 320 1,09 1,18 1,04 1,31 160 1,20 1,29 1,60 1,20 1,09 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,00 1,50 1,50 1,50 1,50 1,50 1,50 1,50 1,50									
2005 160 0,93 1,16 2009 160 1,05 0,84 2009 160 1,05 0,98 2009 320 1,09 1,18 2005 160 1,20 1,29 2009 160 0,92 0,94 2009 160 0,92 0,94 2009 320 0,99 0,97 320 0,99 0,97 3200 0,99 0,93 320 0,99 0,97 3200 0,99 0,83		06'0	0,71 0,60		0,54 0,46	0,34	0,24	0,21	0,14
2005 160 0,93 1,16 2009 160 1,05 0,98 2009 160 1,05 0,98 2009 320 1,09 1,18 2005 160 1,20 1,29 2009 160 0,92 0,94 20 2009 320 0,99 0,97 2009 320 0,99 0,97 320 0,99 0,97 320 0,99 0,93		-0,31	16,0- 77,0-	·	-0,93	-1,12	-1,21	-1,18	-1,22
2009 160 1,05 0,98 0,98 0,43 0,10 0 1,05 0,98 0,98 0,98 0,99 0,99 0,99 0,99 0,99		98'0	0,89		0,53 0,36	0,22	0,16	0,14	90'0
2009 160 1,05 0,98   2009 320 1,09 1,18   2005 160 1,20 1,29 1,50   2009 160 0,92 0,94   2009 320 0,94 0,97   2009 320 0,99 0,97   3200 0,99 0,97   3200 0,99 0,97   3200 0,99 0,83   320 0,99 0,99 0,99 0   320 0,99 0,99 0,99 0   320 0,99 0,99 0   320 0,99 0,99 0   320 0,99 0,99 0   320 0,99 0,99 0   320 0,99 0,99 0		-0,33	-0,21 -0,36		-0,67 -0,83	-0,94	-0,95	-0,91	-0,95
2009 320 1,09 1,18  2005 160 1,20 1,29  2009 160 0,92 0,94  2009 320 0,99 0,97  2009 320 0,99 0,97  3200 0,99 0,83  320 0,13 -0,23		0,61	0,39 0,29		0,25 0,24	0,13	80'0	80'0	0'0
2009 320 1,09 1,18  2005 160 1,20 1,29  2009 160 0,92 0,94  2009 320 0,99 0,97  2009 320 0,99 0,97  3 0,05 160 0,99 0,83  3 0,07 -0,13 -0,85	·	68'0-	-1,13 -1,15	·	-1,07 -0,99	-1,05	-1,03	-0,97	-0,94
2009 320 1,09 1,18  2005 160 1,20 1,29  2009 160 0,92 0,94  2009 320 0,99 0,97  2009 320 0,99 0,97  3200 0,99 0,83  320 0,13 -0,23									
2005 160 1,20 1,29 2009 160 0,92 0,94 2009 320 0,99 0,97 2009 320 0,99 0,97 3200 0,99 0,97 3200 0,99 0,83 320 0,99 0,83	_	1,45	1,30 1,1	,14 0,	0,95 0,87	0,74	0,62	0,61	0,53
2005 160 1,20 1,29 2009 160 0,92 0,94 2009 320 0,99 0,97 3200 0,99 0,97 3200 0,99 0,83 320 0,99 0,83 320 0,99 0,83		1,47	0,79 0,31	11,0-	11 -0,24	-0,45	-0,61	-0,59	-0'67
2009 160 0,92 0,94 2009 320 0,94 2009 320 0,99 0,97 2005 160 0,99 0,83 2005 2005 160 0,99 0,83		1,39	1,34 1,7	1,14 0,	99'0 62'0	0,40	0,28	0,23	0,14
2009 160 0,92 0,94  2009 320 0,99 0,97  2005 160 0,99 0,83  2005 160 0,99 0,83		0,91	0,64 0,22		-0,30 -0,44	-0,72	-0,81	-0,82	98'0-
2009 320 0,99 0,97 5.005 160 0,99 0,83 c.0,13 -0,86		0,83	0,55 0,37		0,38 0,40	0,24	0,13	80'0	80'0
2009 320 0,99 0,97 S -0,07 -0,23 2005 160 0,99 0,83 S -0,13 -0,85	·	-0,39	-0,84 -1,01		82'0- 68'0-	-0,91	86'0-	-0,97	-0,93
320 0,99 0,97 -0,07 -0,23 160 0,99 0,83 -0,13 -0,85									
-0,07 -0,23 160 0,99 0,83 -0,13 -0,85 -		66'0	0,94 0,96		0,85 0,80	0,84	0,73	0,73	69'0
160 0,99 0,83 -0,13 -0,85		-0,04	-0,17 -0,10		-0,37	-0,27	-0,43	-0,40	-0,44
-0,13 -0,85		0,50	0,38 0,29		0,25 0,21	0,18	0,12	0,12	60'0
		-1,15	-1,16 -1,13		-1,08 -1,02	66'0-	66'0-	-0,93	-0,92
. 1986/2006-1999/2009 160 0,99 1,00 0,86		0,59	0,47 0,27		0,16 0,07	0'0	0'02	0,05	0,04
Test Statistics -0,05 0,01 -0,46		-0,95	-0,98 -1,17		-1,21 -1,21	-1,12	-1,07	-1,01	-0,97

\* Test Statistics in bold are significant at 90% level of confidence

Table 3C Variance Ratios for Monthly Stock Index Returns the World and Different Countries (Continuation)

Sample Period	Obsev.		Numb	er "q" (lags mont	hs) of base observ	ations aggregate	Number "q" (lags months) of base observations aggregated to form the variance ratio	ance ratio					
	"bu"	3	9	12	24	36	48	09	72	84	96	108	120
9. RLNL V. W. Index													
. 1973/2002-1999/2009	320	1,09	1,10	26'0	0,75	0,65	0,58	0,55	0,49	0,39	0,29	0,23	0,13
Test Statistics		1,12	0,70	-0,14	-0'80	-0,92	-0,94	06'0-	-0'95	-1,04	-1,13	-1,16	-1,23
. 1973/2002-1986/2005	160	1,04	1,13	1,12	98'0	0,70	0,58	0,47	0,39	0,27	0,20	0,18	60'0
Test Statistics		0,32	0,64	0,40	-0,32	-0,55	-0'68	-0'16	-0,79	-0'88	06'0-	-0,87	-0,92
. 1986/2006-1999/2009	160	1,13	1,04	08'0	95'0	0,49	0,40	0,34	0,30	0,19	0,13	60'0	0,04
Test Statistics		1,14	0,22	99'0-	-1,01	96'0-	96'0-	-0,94	-0,91	-0'97	86'0-	-0,97	76'0-
10. RLUK V. W. Index													
. 1973/2002-1999/2009	320	1,04	1,04	0,88	0,32	0,31	0,24	0,24	0,22	0,20	0,19	0,18	0,16
Test Statistics		0,44	0,27	-0,58	-2,21	-1,81	-1,72	-1,55	-1,44	-1,36	-1,29	-1,23	-1,20
. 1973/2002-1986/2005	160	1,06	1,15	1,03	0,30	0,26	0,16	0,11	0,10	80'0	80'0	80'0	0'02
Test Statistics		0,54	9/'0	0,12	-1,62	-1,38	-1,35	-1,27	-1,17	-1,11	-1,04	-0,98	-0'95
. 1986/2006-1999/2009	160	0,95	0,78	0,51	0,17	0,17	60'0	60'0	0'0	90'0	0,04	0'03	0,02
Test Statistics		-0,42	-1,14	-1,66	-1,93	-1,54	-1,46	-1,30	-1,22	-1,14	-1,08	-1,03	66'0-
11. RLUS V. W Index													
. 1973/2002-1999/2009	320	0,97	0,94	68'0	65'0	0,48	0,47	0,43	0,33	0,28	0,26	0,22	0,14
Test Statistics		-0'39	-0,42	-0,53	-1,34	-1,37	-1,21	-1,15	-1,23	-1,22	-1,19	-1,17	-1,23
. 1973/2002-1986/2005	160	66'0	1,11	1,08	0,52	0,31	0,31	0,27	0,25	0,15	0,12	0,11	0,04
Test Statistics		-0,11	0,59	0,26	-1,10	-1,28	-1,10	-1,05	86'0-	-1,02	66'0-	-0,94	96'0-
. 1986/2006-1999/2009	160	0,92	0,70	95'0	0,39	0,40	0,34	0,24	0,16	0,14	0,11	0'02	0,04
Test Statistics		-0'67	-1,53	-1,50	-1,40	-1,12	-1,05	-1,08	-1,09	-1,03	-1,00	66'0-	-0,97

\* Test Statistics in bold are significant at 90% level of confidence

What do we see in Figures 2 to 12 in the appendix? Two main conclusions can be drawn: it seems that only in the first sub-period (1973–1986) is there evidence of short term positive autocorrelation with the timing of the mean reversion varying across countries. However, it seems that the second sub-period (1986–1999) does not reveal any short-term mean reversions. Again, one must remember that some variance ratios do not have statistical power. Hence, an additional approach to corroborate these observations should be found.

Figure 3 Variance Ratios of the Canadian Value Weighted Stock Market Index

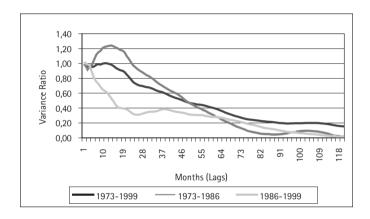


Figure 4 Variance Ratios of the Austrian Value Weighted Stock Market Index

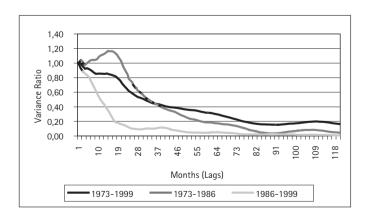


Figure 5 Variance Ratios of the Swiss Value Weighted Stock Market Index

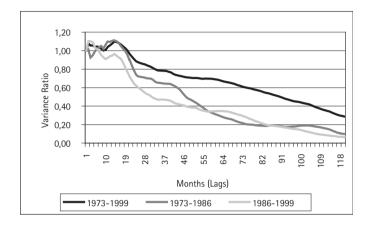


Figure 6 Variance Ratios of the French Value Weighted Stock Market Index

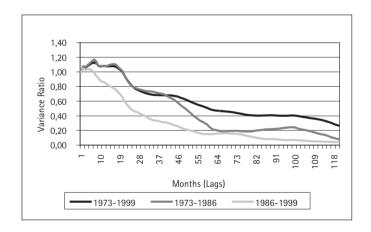


Figure 7 Variance Ratios of the German Value Weighted Stock Market Index

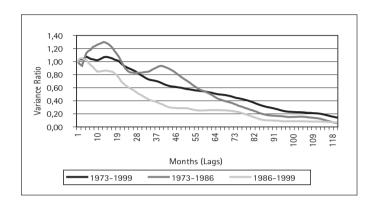


Figure 8 Variance Ratios of the Italian Value Weighted Stock Market Index

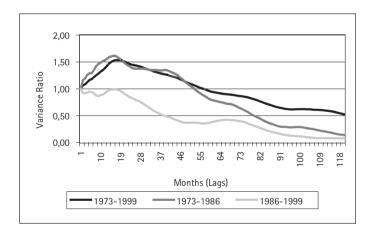


Figure 9 Variance Ratios of the Japanese Value Weighted Stock Market Index

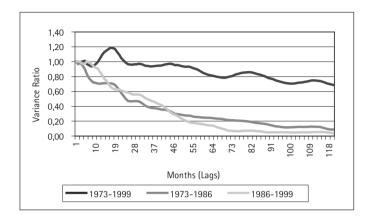


Figure 10 Variance Ratios Of The Dutch Value Weighted Stock Market Index

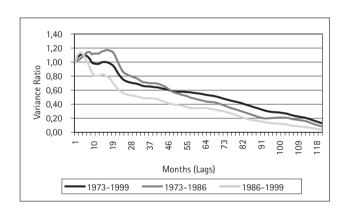


Figure 11 Variance Ratios of the British Value Weighted Stock Market Index

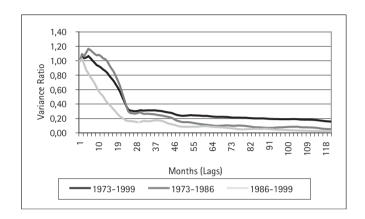
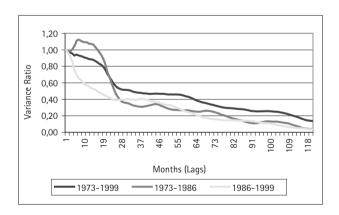


Figure 12 Variance Ratios of the Us Value Weighted Stock Market Index



If we compare part B and part C in table 3 for the whole period (1973–1999), it is interesting to see the differences in timing of short-run overreaction and mean reversion. In the case of Italy, the stock index reveals overreaction in one year and in the case of Great Britain the stock index shows mean reversion in two years for the whole period.

One should not be surprised by the evidence of positive and negative serial correlation. These results are supported by previous studies, e.g. Poterba and Summers (1988):

Our results suggest that stock returns show positive serial correlation over short periods and negative correlation over longer intervals. This conclusion emerges from data on equal-weighted and value-weighted NYSE and is corroborated by data from other nations and time periods...

From the empirical evidence reported in table 3, we may conclude the presence of the two types of serial correlation for the value-weighted stock index returns and the presence of positive serial correlation in short time intervals for the value-weighted Peruvian stock index returns. The apparent mean reversion could be caused by the presence of transitory components in stock prices as opposed to permanent components (random walk).

# 4. THE LEAD-ON-THE-LAG REGRESSION APPROACH

#### 4.1 Theoretical framework

A plausible explanation for the positive serial correlation in the short run and the subsequent negative serial correlation is the combination between overreaction and mean reversion. The presence of both phenomena cast some doubt on the pure random walk behaviour of stock prices. In the previous section we saw some traces of overreaction and mean reversion, but were not able to test for long run mean reversion.

Now, we are interested in knowing why this phenomenon may occur. Perhaps there are transitory components that in some way are stationary processes and this could lead to short run deviations from the fundamental price under market efficiency conditions. For example, let's consider the following process (Fama and French, 1988):

$$p_t = p_t^P + p_t^T (20)$$

$$p_t^P = \mu + p_{t-1}^P + \varepsilon_{1t} \tag{21}$$

$$p_t^T = stationary.process$$
 (22)

Equation 20 represents the observed price in the stock market. hawse have assumed so far that a pure random walk process drives this price, but this could be wrong if there is a permanent component (random walk) and a transitory component (zero mean stationary process). Equation 21 simply holds that the permanent component follows a random walk process, while equation 22 holds that the transitory component is in fact an stationary process and in this sense subject to some degree of predictability.

Alternatively, one may see equation 21 as the fundamental stock price in efficient markets, while equation 22 represents the short run deviation from its fundamental value. Due to the fact that we are interested in returns instead of prices, one could also define the lead and the lag *returns* in terms of the permanent and transitory components:

$$r_{t+q}^{q} = p_{t+q} - p_{t} = p_{t+q}^{P} - p_{t}^{P} + p_{t+q}^{T} - p_{t}^{T}$$
(23)

$$r_{t-q}^{q} = p_{t} - p_{t-q} = p_{t}^{P} - p_{t-q}^{P} + p_{t}^{T} - p_{t-q}^{T}$$
(24)

Equation 23 represents the lead return, while equation 24 represents the lag return. One approach to see the behaviour of both components along the time is by conducting a regression of the lead return on a constant and the lag return. Likewise, one can establish the first order autocorrelation coefficient of the "q" period changes in the transitory component:

$$\rho_{1}^{q} = \frac{Cov(p_{t+q}^{T} - p_{t}^{T}, p_{t}^{T} - p_{t-q}^{T})}{\sigma^{2}(p_{t+q}^{T} - p_{t}^{T})}$$

$$\rho_{1}^{q} = \frac{-\sigma^{2}(p_{t}^{T}) - Cov(p_{t}^{T}, p_{t+2q}^{T}) + 2Cov(p_{t}^{T}, p_{t+q}^{T})}{2\sigma^{2}(p_{t}^{T}) - 2Cov(p_{t+q}^{T}, p_{t}^{T})}$$
(25)

Due to the fact that the transitory component is a zero mean stationary process, the covariances become zero when the "q" interval is very large. In such a case, it is not difficult to show that the first order autocorrelation coefficient of the transitory component will tend to –0,5 in equation 25. Now, let's consider the lead-on-the-lag regression approach first stated by Fama and French (1998):

$$r_{t+q}^{q} = c(q) + \beta(q)r_{t-q}^{q} + \varepsilon_{t+q}^{q}$$
 (26)

Taking into account equations 23 and 24, the slope in the lead-on-the-lag regression is equal to the following expression:

$$\beta(q) = \frac{Cov(r_{t+q}^{q}, r_{t-q}^{q})}{\sigma^{2}(r_{t-q}^{q})} = \frac{Cov(p_{t+q}^{P} - p_{t}^{P} + p_{t+q}^{T} - p_{t}^{T}, p_{t}^{P} - p_{t-q}^{P} + p_{t}^{T} - p_{t-q}^{T})}{\sigma^{2}(p_{t}^{P} - p_{t-q}^{P}) + \sigma^{2}(p_{t}^{T} - p_{t-q}^{T})}$$

$$\beta(q) = \frac{\rho_1^q \sigma^2 \left( p_{t+q}^T - p_t^T \right)}{\sigma^2 \left( p_t^P - p_{t-q}^P \right) + \sigma^2 \left( p_t^T - p_{t-q}^T \right)}$$
(27)

In expression 27, when the stock price is driving just by the transitory component, the variance of the permanent component in equation 27 will be zero and the slope will be equal to the first-order autocorrelation coefficient of the "q" period (expression 25). Furthermore, we know from equation 25 that in this case the first order autocorrelation coefficient will approach to -0,5 for increasing horizons (q). Therefore, the slope must approach -0,5 for increasing horizons whenever only transitory components are present.

$$\beta(q) = Lim_{q \to \infty} \rho_1^q = -0.5 \tag{28}$$

From equation 27, we can also foresee what would happen if prices were driven only by permanent components (pure random walk). In such a case, the variances of the transitory components will be zero as well as the slope of the regression. The most interesting question is what happens if the stock price includes both components. In such a case the slopes will take a "U" shape when plotted against the time interval "q".

The slopes will start from zero as they are dominated in the short run by the random walk component to then become negative as the transitory component dominates the permanent one. Finally, they will return to zero as the transitory component is again dominated by the random walk component in the long run. In this sense, this approach could be very helpful for testing mean reversion in longer time horizons.

# 4.2 Empirical evidence

This section shows the empirical evidence corresponding to the value-weighted stock index for ten countries and the world index. Table 4 (A and B) highlights two facts: slopes above zero in the very short run and negative slopes in the long run which tend to come back to

zero as the time interval "q" increases. Furthermore, the results are significant for the world and for several countries as well. However, the accuracy of our estimations decreases as the time interval increases, for example if one looks at Germany, the slope in the three months calculation have a standard error of 0,06, this increases to 0,25 in the ten-year horizon. This leaves too much uncertainty to deal with although the slopes are significant.

From figures 13 to 24, one can see the "U" shape of the slopes when they are plotted against the time interval. The existence of such pattern points to a plausible hypothesis of the presence of both components in stock prices. Furthermore, in all cases they start above zero in the very short term (3 months), pointing to a short run overreaction or mean reversion.

Figure 13
Ols Slopes for the World Value-weighted Stock Market Index

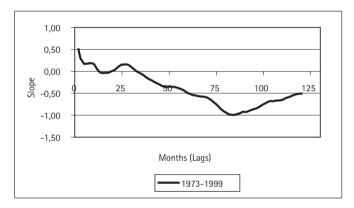


Figure 14
Ols Slopes for the Canadian Value-weighted Stock Market Index

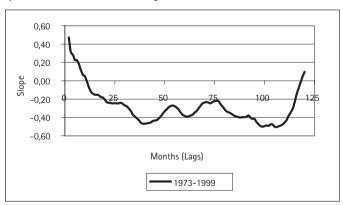


Table 4A Ols Slopes for Monthly Stock Index Returns. The World And Different Countries

1973/2002-1999/2009 3 6  1. World V.W.Index Slope (B) 0,37 0,18 Svd(B) 0,07 0,13 Svd(B) 0,07 0,12 Slope (B) 0,31 0,22 Svd(B) 0,07 0,12 T-Statistic 4,71 1,93 3. RLAU V.W. Index Slope (B) 0,07 0,11 T-Statistic 3,98 1,04 4. RLSW V.W. Index Slope (B) 0,30 0,17 Svd(B) 0,06 0,11 T-Statistic 4,85 1,64 Svd(B) 0,36 0,17 Svd(B) 0,36 0,17 Svd(B) 0,36 0,17 Svd(B) 0,06 0,11 T-Statistic 5,65 1,52 T-Statistic 5,65 1,52	0,04 0,12 0,12 0,35 -0,07 0,13 -0,54 -0,54	0,13 0,09 1,54 -0,25 0,14 -1,74	-0,08 0,11 -0,71	48	09	72	84	96	108	120
c 4,29 ex 0,31 ex 0,31 ex 0,31 ex 0,30 ex 4,71 ex 0,30	0,04 0,12 0,12 0,35 -0,07 0,13 -0,54 -0,54	0,13 0,09 1,54 -0,25 0,14 -0,28 0,12	-0,08 0,11 -0,71 -0,41							
c 4,29 ex 0,31 ex 0,31 ex 0,32 ex 0,30	0,04 0,12 0,13 0,13 0,13 -0,54 -0,54 0,10	0,13 0,09 1,54 -0,25 0,14 -1,74 -0,28 0,12	-0,08 0,11 -0,71							
ex 4,29 ex 0,31 c 4,71 c 4,71 ex 0,30 c 3,98 lex 0,30 c 0,06 c 5,65	0,12 0,35 -0,07 0,13 -0,54 -0,07 0,10	0,09 1,54 -0,25 0,14 -1,74	0,11	-0,35	-0,49	-0,65	66'0-	-0,85	-0,66	-0,51
ex 0,31 c 4,29 cx 0,31 cx 0,32 cx 3,98 cx 0,30 cx 0,30 cx 0,36	-0,07 -0,07 0,13 -0,54 -0,07 0,10	-0,25 0,14 -1,74 -0,28 0,12	-0,71	60'0	60'0	0,13	80'0	90'0	0,04	0'02
c 4,71 c 4,71 c 4,71 c 4,71 c 4,71 c 4,85 c 3,98 c 0,30 c 0,66 c 0,66 c 5,66	-0,07 0,13 -0,54 -0,07 0,10 -0,64	-0,25 0,14 -1,74 -0,28 0,12	-0,41	-3,84	-5,42	-5,16	-12,45	-14,16	-15,97	-10,00
C 4,71  C 4,71  (ex 0,29  0,07  0,30  0,06  0,06  0,35  0,35  0,35	-0,07 0,13 -0,54 -0,07 -0,07 -0,64	-0,25 0,14 -1,74 -0,28 0,12	-0,41							
c 4,71 cx 0,29 c 3,98 cc 3,98 cc 0,30 c 0,06 c 6,35 c 6,66	-0,13 -0,54 -0,07 -0,10 -0,64	0,14 -1,74 -0,28 0,12		-0,39	-0,38	-0,24	-0,37	-0,45	-0,49	60'0
ex 0,29 (0,29)(20)(20(20(0,29 (0,29 (0,29 (0,29 (0,29 (0,29 (0,29 (0,29 (0,29)	-0,54 -0,07 0,10 -0,64	-1,74 -0,28 0,12	80'0	60'0	80'0	60'0	0,11	0,10	90'0	60'0
C 0,29 0,077 0,077 0,30 0,30 0,06 0,06 0,06 0,06 0,06 0,06	-0,07	-0,28 0,12	-4,97	-4,36	-4,51	-2,73	-3,30	-4,54	-10,12	1,06
0,29 0,72 0,07 0,07 0,07 0,30 0,06 0,06 0,06 0,06	-0,07	-0,28 0,12								
13.98 (c. 3.98 (c. 3.	0,10	0,12	-0,36	-0,43	-0,29	-0,27	-0,80	-0,67	-0,61	-0,65
198 (c 3.98 (c) 6.30 (c) 6.35	-0,64		0,10	80'0	60'0	0,13	0,16	0,11	80'0	90'0
0,30 0,06 0,06 0,35 0,35 0,06 5,65	2	-2,42	-3,56	-5,34	-3,17	-2,11	-5,01	-6,11	-7,65	-11,04
0,30 0,06 0,06 0,35 0,06	200									
0,06 4,85 0,35 0,06 5,65	40,0	00'0	0,12	-0,02	-0,24	-0,34	0,02	0,24	0,17	0,01
0,35 0,06 0,06 0,06	0,10	0,13	0,15	0,19	0,18	0,16	0,11	0,14	0,13	0,14
0,35 0,06 0,06 5,65	0,40	-0,04	0,78	-0,13	-1,35	-2,13	0,21	1,65	1,27	80'0
0,35 0,06 5,65										
0,06	-0,04	-0,12	-0,15	-0,18	-0,30	-0,58	-0,92	-0,80	-0,68	-0,54
5,65	60'0	0,15	60'0	60'0	0,10	0,11	60'0	0,07	0,05	0,04
200	-0,45	-0,80	-1,67	-2,03	-2,96	-5,17	-10,71	-11,23	-12,57	-12,13
6. RLGE V. W. Index										
Slope (B) 0,36 0,14	0,01	-0,25	-0,24	-0,51	-0,62	-0,81	-0,80	-0,51	-0,45	92'0-
Std(B) 0,06 0,09	0,11	0,16	0,11	0,11	0,10	90'0	0,07	0,15	0,16	0,25
T-Statistic 6,24 1,67	0,13	-1,54	-2,28	-4,52	-6,10	-12,58	-11,55	-3,47	-2,79	-3,01
7. RLIT V.W. Index										
	0,17	90'0-	-0,13	-0,26	-0,23	-0,57	-0,84	-0,64	-0,49	-0,47
Std(B) 0,07 0,11	0,12	0,12	0,10	0,11	0,12	0,10	90'0	90'0	0,05	0'0
T-Statistic 6,02 2,90	1,42	-0,51	-1,39	-2,47	-2,02	-5,87	-14,46	-12,43	-10,02	-6,52

\* Test Statistics in bold are significant at 90% level of confidence

Table 4B Ols Slopes for Monthly Stock Index Returns the World and Different Countries (Continuation)

Sample Period			The l	ead on the lag"	regression mod	The lead on the lag" regression model: $r(t,t+q)=c(q)+B(q)r(t-q,t)+e(t,t+q)$	(q) + B(q)r(t-q,t	) + e(t, t+q)				
1973/02-1999/09	က	9	12	24	36	48	09	72	84	96	108	120
8. RLIP V.W. Index												
Slope (B)	0,35	0,21	0,17	0,41	0,21	90'0	80'0-	-0,46	-1,24	-1,81	-1,40	-1,25
Std(B)	90'0	0,10	0,10	0,11	0,13	0,14	0,17	0,25	0,26	0,12	60'0	0,12
T-Statistic	7,02	2,05	1,73	3,60	1,68	0,34	-0,48	-1,86	-4,78	-14,51	-16,27	-10,61
9. RLNL V. W. Index												
Slope (B)	0,34	80'0	0,01	-0'03	-0'03	-0,34	-0,59	-0,78	-0,79	-0,52	-0,30	-0,12
Std(B)	90'0	0,11	60'0	80'0	0,13	0,11	0,10	0,05	20'0	0,11	60'0	0,10
T-Statistic	5,46	0,71	0,15	-0,39	-0,25	-3,06	-6,14	-14,94	-12,15	-4,69	-3,52	-1,24
10. RLUK V. W. Index												
Slope (B)	0,33	0,03	-0,31	00'0	-0'03	-0,02	-0,13	-0,24	-0,61	09'0-	-0,48	-0,37
Std(B)	80'0	0,18	0,12	0,13	60'0	0,10	0,13	0,14	0,18	0,16	0,13	0,10
T-Statistic	4,02	0,18	-2,62	-0,03	-0,36	-0,18	-1,00	-1,72	-3,37	-3,78	-3,65	-3,50
11. RLUS V. W Index												
Slope (B)	0,30	0,15	-0,07	0,01	0,10	-0,02	-0,31	-0,30	-0,27	-0,17	-0,12	0,29
Std(B)	20'0	0,12	0,10	0,12	0,17	0,12	0,11	0,11	60'0	0,11	0,10	0,11
T-Statistic	4,07	1,27	-0,72	0,05	0,62	-0,15	-2,82	-2,82	-2,95	-1,50	-1,19	2,70

\* Test Statistics in bold are significant at 90% level of confidence

Figure 15 Ols Slopes for the Austrian Value-weighted Stock Market Index

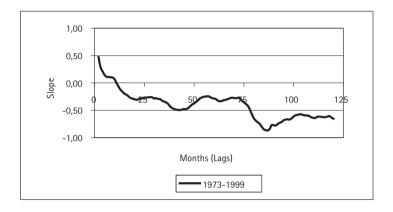


Figure 16
Ols Slopes for the Swiss Value-weighted Stock Market Index

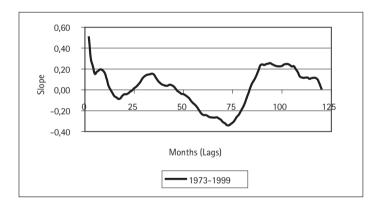


Figure 17 Ols Slopes for the French Value-weighted Stock Market Index

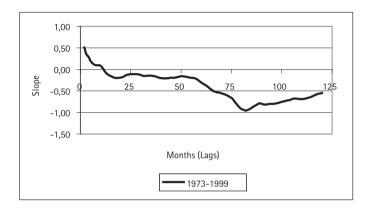


Figure 18
Ols Slopes for the German Value-weighted Stock Market Index

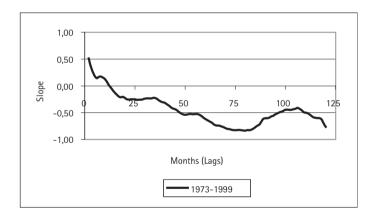


Figure 19
Ols Slopes for the Italian Value-weighted Stock Market Index

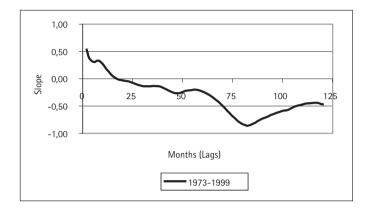
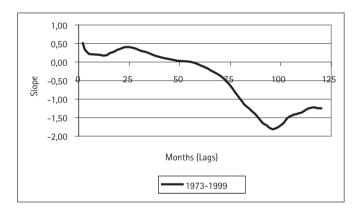


Figure 20 Ols Slopes for the Japanese Value-weighted Stock Market Index





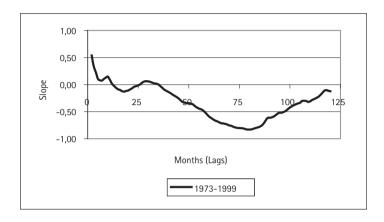
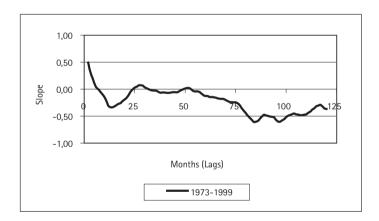


Figure 22 Ols Slopes for the British Value-weighted Stock Market Index





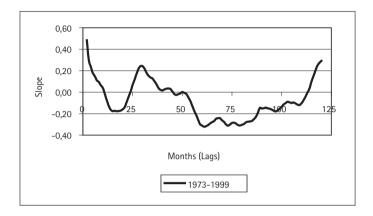
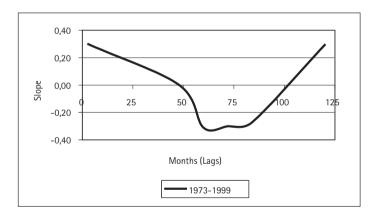


Figure 24
Ols Significant Slopes for the US Value-weighted Stock Market Index



In any case, it does not seem a random walk pattern in the short run. However, one also finds non-significant slopes in the one and two year time intervals, so one cannot be conclusive in the precise shape of the "U". Besides, apparently some countries do not come back to zero in the long run and others, like Switzerland, seem to complete their cycle and start a new one. A plausible explanation for this result could be the apparent differences in timing for overreaction and mean reversion across countries.

An important departure from previous studies is the significance of the results found, perhaps due to the fact that the slopes were not adjusted by the 'true slope bias'. This bias arises because we do not know the true price generating process that equals slopes to zero. Therefore, we must adjust the slopes in order to take this bias into account.

The problem is usually solved using simulations of different alternative random walk models that can give us true slopes equal to zero. Although, we didn't perform such task, the estimates do take into account the artificial autocorrelation problem that is generated in obtaining the lead and the lag overlapping returns. The problem of artificial serial correlation was corrected using the Newey-West Heteroscedasticity Consistent Covariance (HAC). The use of the Newey-West HAC is also consistent with the third version of the random walk model.

#### CONCLUSION

So far hawse have seen evidence of the presence of transitory and permanent components in stock prices. From the results, we can see that there are some traces of overreaction and mean reversion in the short run and mean reversion in the long run. However, one cannot be conclusive about long-term mean reversion, although we may see some traces of it.

The results also point to timing and duration differences in the emergence of market anomalies across countries, probably due to differences in investment horizons across capital markets. Unfortunately little is known about what drives such differences.

Could one conclude that the stock market is not efficient or saying that the random walk hypothesis does not hold? In general, one cannot reject the efficient market hypothesis (EMH) just by saying that one of its implications does not hold. The heart of the EMH is the rational behavior of investors, but sometimes "rationality" could imply "irrationality" in the sense that people do something irrational because they really believe that this action will increase their well-being, and for example overreact because they expect for more profits.

According to Fama (1998) overreaction and mean reversion could be consistent with the EMH and can occur in an efficient capital market to the extent that they cannot be predicted. Hence, a random sequel of overreaction and mean reversion could be consistent with changes in market equilibrium.

This explanation seems plausible, but it is not the only one. According to Haugen (1999), investors could anticipate overreaction and mean reversion, so these anomalies truly reflect market inefficiencies in the short run. These observations point towards short run market *inefficiency* and long run market *efficiency*.

In other words, long-run mean reversion could be compatible with the EMH because this could represent time-varying long-run market equilibrium. In the short run, it is unlikely that random walk forces dominate transitory ones. Therefore, there could be positive autocorrelation and overreaction that would be quickly reversed. In the short run there is a chance that stock markets depict investors' behavior that is not consistent with EMH, while in the long run it is more likely that the stock market will show a picture closer to the EMH.

The implications of long run market efficiency are several: it implies that a buy-and-hold strategy is better than speculating in the short run, that one could use equilibrium models (such as the Capital Asset Pricing Model-CAPM) for evaluating long-term capital expenditures proposals, and so on. Unfortunately, one still needs to develop better econometric tools in order to be more conclusive about the magnitude of these anomalies.

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